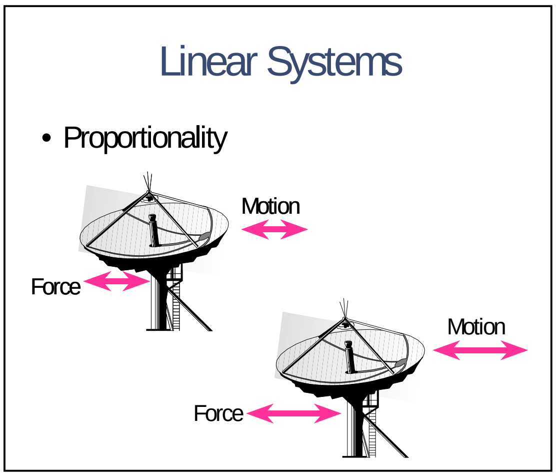
An Introduction to Linear and Non Linear Systems And their Relation to Machinery Faults

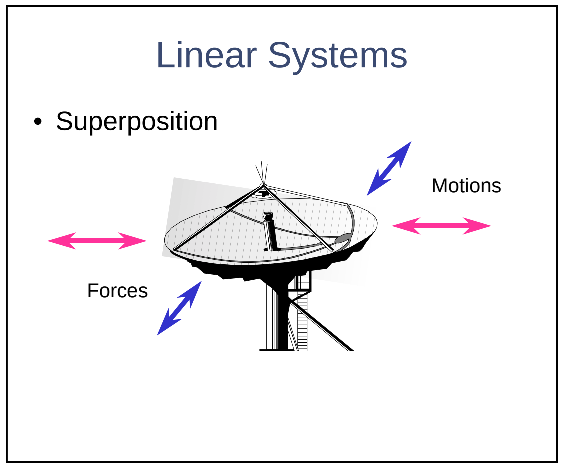
By Alan Friedman

Introduction

This paper is meant to introduce the reader to the concepts of linear and non-linear systems in the context of machinery vibration analysis by describing their relationship to machine health. When one analyzes the vibration spectrum of a machine in the context of linearity and non-linearity, one may arrive at a better understanding of why spectra look as they do and how the appearance of a spectrum relates to machine health. This paper will be written in simple language in order to be understood by those without technical backgrounds



(Figure 1)

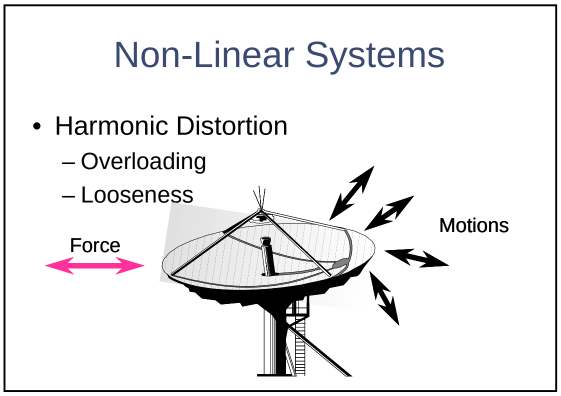


(Figure 2)

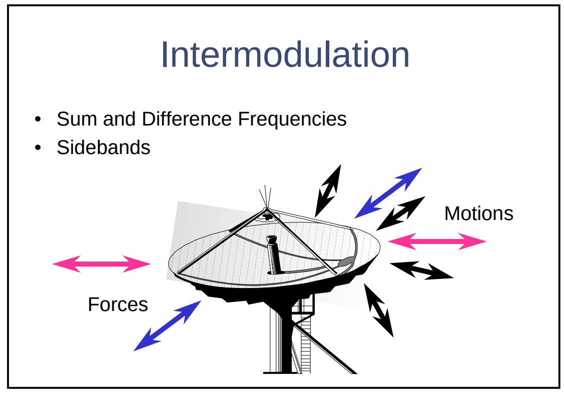
Linear Systems

If one considers a linear system a black box, one can say that what comes out of the box is directly proportional to what goes in. This is coincidentally called "proportionality" and is described in Figure 1 above. Here we can see that the output motion is directly related to the input force. If the input force increases, the resulting motion also increases proportionally.

"Superposition" is another quality of linear systems as demonstrated in Figure 2 above. Superposition means that if we have 2 or more input forces, the output motion will be proportional to the sum of the input forces. In other words, nothing new is created. If we add a whole bunch of forces at the input, the output motion will still be directly proportional to the sum of those forces.



(Figure 3)

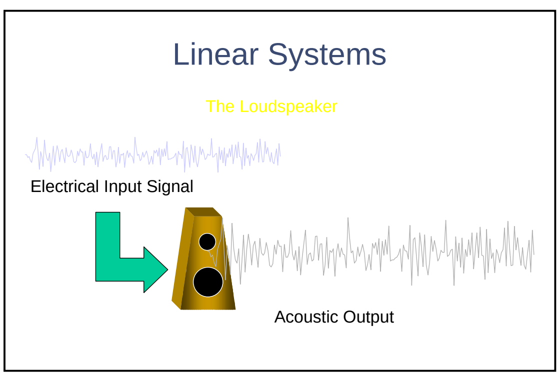


(Figure 4)

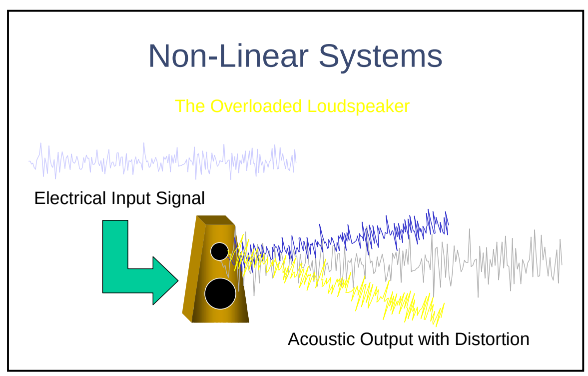
Non-Linear Systems

Consider a dense metal cube sitting on some ice. If you push the cube, it will slide proportionally to how hard you push it. This is a linear response. Now consider that the cube is made out of gelatin. When you give the gelatin a push it may slide a bit, but it will also wiggle and wobble all over the place. This is an example of a non-linear response. The gelatin doesn't move only in the direction of the push, it also wiggles around in a whole bunch of different directions. Therefore we can say that the output motion is not directly proportional to the input force and therefore the gelatin block is non-linear. Figure 3 above demonstrates this principal.

Non-Linear systems also don't follow the law of Superposition. This means that the output response is not proportional to the sum of the input forces. In a non-linear system, the inputs combine with each other and produce new things in the output that were not present in the input. (Figure 4)



(Figure 5)



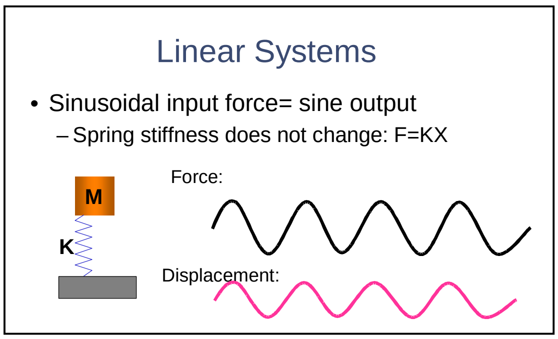
(Figure 6)

Here is another example of linear and non-linear systems that should be easy to relate to. The example is demonstrated by figures 5 and 6. When one plays a stereo at a relatively low volume, the music comes out clearly. If one raises the volume slightly, the music comes out of the speaker more loudly, but still sounds good. This is a linear response.

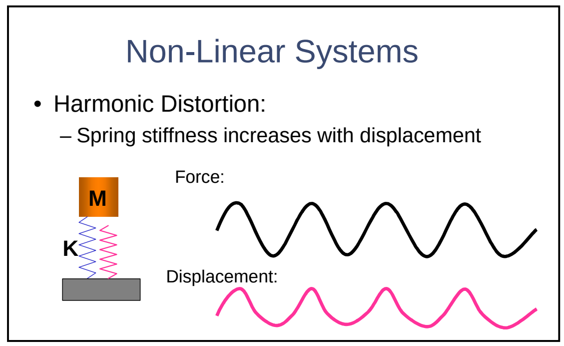
We reach a point however where if we make the stereo loud enough, the music becomes all distorted and we begin to hear new sounds that were not recorded on our CD! This is a non-linear response. The key again to understanding when something is non-linear is that the output contains things that were not present in the input.

Linearity and Non-Linearity in Vibration

Now that we have described the basic concepts of linearity and non-linearity, it is time to discuss them in terms of vibration signals. Simple mass-spring systems as shown in figures 7 and 8 will be used for this discussion.



(Figure 7)



(Figure 8)

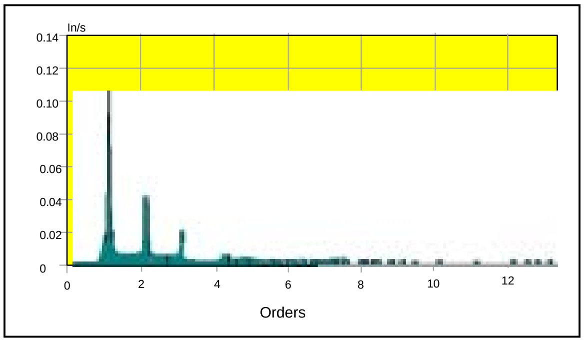
In figure 7, we have an ideal mass / spring system that can be described by the equation F = KX where "F" is the input force, "K" is the spring stiffness and "X" is the resulting displacement of the spring. This is a linear system. If we input a sinusoidal force, the resulting displacement is also sinusoidal and proportional to the input.

In Figure 8, the stiffness of the spring changes as it is stretched and compressed. This is a non-linear system. When we input a sinusoidal force, the resulting displacement is not sinusoidal. This again obeys the rules of nonlinear systems in that we get out something that looks different from what we put in.

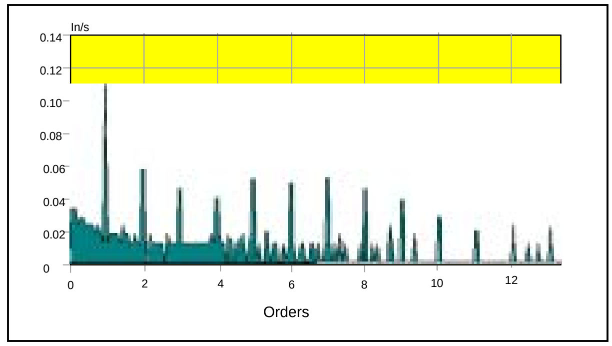
If we remember our basic rules of vibration and the Fast Fourier Transform, the displacement sine wave in figure 7 will produce a single peak in a vibration spectrum. The displacement wave in Figure 8 will produce a peak in the spectrum with harmonics (multiples). This brings us to another important point. The harmonics in this case, are the result of non–linearity.

Machinery Vibration

When we look at the vibration spectra for a machine in the context of linear and nonlinear systems, we can make a very general statement that as machines deteriorate and develop faults they become less linear in their responses. We can also say that many machine faults create non-linearity. Therefore, also in very general terms, we can expect the spectra from a healthy machine to be relatively simple compared to the spectra from a machine with faults. If we consider mechanical looseness as a common machine problem, we can demonstrate this. When the machine is not experiencing looseness and is in good health, its spectra may look like Figure 9.



(Figure 9)



(Figure 10)

In this figure (9), we can see the shaft rate peak (the big one on the left) and a couple of harmonics of the shaft speed. Figure 10 is data from the same machine when it has a looseness problem. What we can see in Figure 10 is that the shaft rate harmonics are both more numerous and higher in amplitude. This is very similar to the example of the two mass-spring systems in that when the mass-spring system was linear, only 1 peak was produced in the spectrum i.e. the output looked like the input. When the mass spring system was non-linear, the output waveform was not sinusoidal and therefore produced harmonics in the spectrum

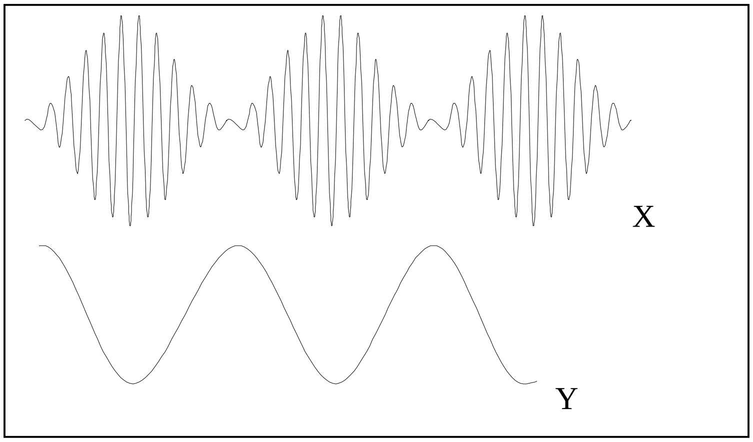
If we take a step back, we can consider that the mechanical input forces in a simple rotating machine are coming from the rotating shaft. If the shaft is rotating perfectly (i.e. there is no looseness) and the response of the machine structure is perfectly linear then we would expect to see only a single peak in our spectrum corresponding to the shaft rate. In other words, the output would look like the input. No machines are perfect however, and shafts do not typically rotate perfectly around their centers and this is why we expect to see some harmonics in machine spectra (as in Figure 9). However, as the machine becomes more nonlinear, due to looseness perhaps, we get more harmonics with higher amplitudes (as in Figure 10).

Note that if one views a spectrum with a linear amplitude scale, one may not see the harmonic content of the spectrum if the harmonics are much smaller in amplitude than the shaft rate peak. If one views the same data, using a logarithmic amplitude scale, more harmonic content will be visible on the graph.

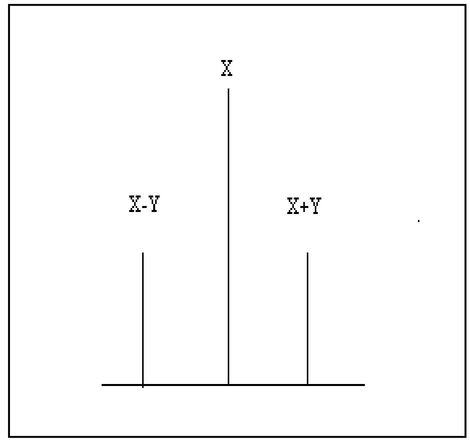
Looseness, Foundation Cracks and Broken Mounting Bolts can cause non-linearity in machines that may result in harmonics in a spectrum.

Sidebands

Sidebands in a spectrum are another result of non-linearity. Sidebands are produced by amplitude modulation as shown in Figure 11.



(Figure 11)



(Figure 12)

The top waveform in figure 11 is an example of a modulated waveform. What we have here is a wave that repeats itself with a frequency "X", however, the amplitude of this wave goes up and down at the frequency "Y" of the wave on the bottom of the diagram. The bottom wave is simply included to demonstrate the frequency at which the amplitude of the top wave goes up and down.

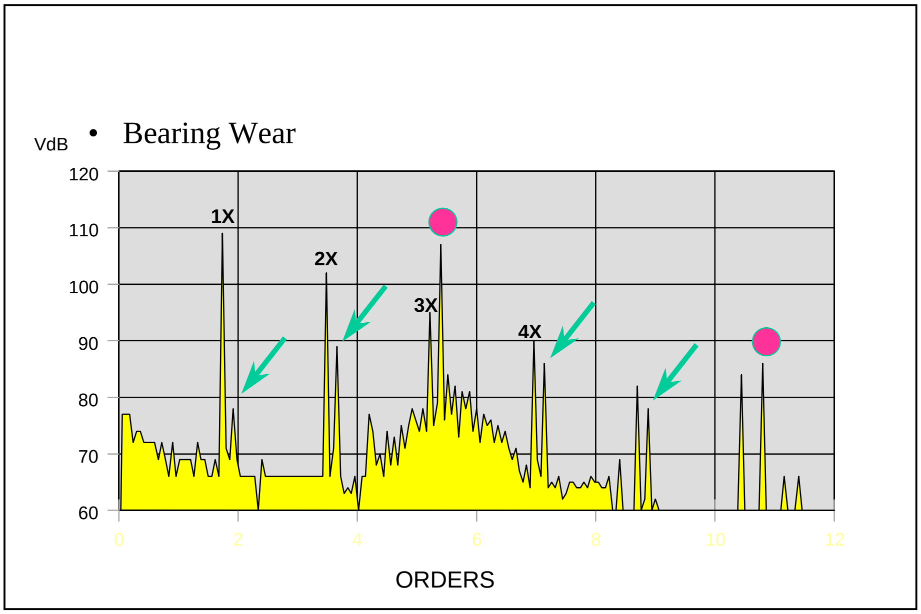
If one wishes to visualize this in mechanical terms, consider a set of gears where one gear is not centered on its shaft. Lets say that the non-centered gear has 32 teeth. In one revolution of the non-centered gear we will see 32 tooth impacts. This would relate to frequency "X" above. Since this gear is not centered on its shaft, the amplitude of the tooth impacts will go up and down as the gear moves closer and farther away from the second gear. It will take one revolution of the non-centered gear for the level of the impacts to go from maximum to minimum and back to maximum again. So, the frequency with which the levels of the impacts change (or are modulated) is the rotation rate of the non-centered gear. This would relate to frequency "Y" above.

If we look at the spectrum of these gears (Figure 12), what we will see is a peak at frequency "X" with one peak on either side of it "Y" distance away. Stated another way, we will see a peak at frequency "X", another at "X+Y" and a third at "X-Y" The peaks at "X+Y" and "X-Y" are called sidebands.

Why is this system non linear? Because "X+Y" and "X-Y" are not found anywhere in the input signal but they do appear in the output. The only thing in the input is "X" or the rate of the teeth impacting. These impacts go up and down in amplitude at a rate "Y", but there is certainly no "X+Y" or "X-Y" in the input.

The off-centered gear may also cause Frequency Modulation because the effective radius of the off center gear changes as it moves closer and farther from the other gear. As the effective radius changes, the *rate* of tooth contact speeds up and then slows down repetitively. Frequency modulation is similar to amplitude modulation in that it also results in sidebands. In Amplitude modulation, the amplitude of the impacts go up and down in level repeatedly, in Frequency Modulation, the *rate* of impacts gets faster and slower repetitively. In this example, both would result in the same pattern in the spectrum.

Rolling element bearing wear, gear defects, and motor-bar defects will all produce sidebands. Rolling element bearings will also create non-synchronous tones. These are new peaks that are not exact multiples (harmonics of) the shaft rate.



(Figure 13)

Figure 13 shows a machine with a serious bearing problem. Compare this to Figure 9 and note the peaks that are not related to the shaft speed (labeled 1x). The two peaks with circles on them are bearing tones and the peaks with the arrows are sidebands. In terms of linear systems, we can say that this spectrum represents a very non-linear response and suggests the machine has faults (which it does).

To understand why rolling element bearings create non-synchronous tones and sidebands, let us consider the case of a horizontal machine with an inner-race bearing fault. As the shaft and inner race spin, a certain number of balls will impact the fault on the inner race and will produce a peak in the spectrum equal to the number of impacts per revolution of the shaft. This peak is called a bearing tone. The number of impacts will almost never be an integral amount. In other words, there will be 3.1 or 4.7 impacts per revolution, but rarely exactly 3 or 5 impacts. Thus, the peaks will not be direct multiples of the shaft rate and are therefore termed “non-synchronous”. The peak marked with a circle in Figure 13, is an example of a bearing tone at 3.1x the shaft rate.

Considering this example further, we can also see that the weight of the shaft will cause the impacts against the fault to be greater in amplitude when the fault is below the shaft. As the fault on the inner race rotates to the top of the shaft, the impacts will be smaller because there is less weight (load) on the fault. In one revolution of the shaft the fault will travel around 1 time, into the load zone, out of the load zone and back into the load zone. Therefore the frequency of the change of amplitude in this case is equal to the shaft rate and this will also coincide with the spacing of the sidebands around the bearing tone. These peaks are labeled with the arrows in figure 13.

A similar phenomenon occurs if there is a fault on a ball or roller. We will see a bearing tone at a frequency equal to the number of impacts the fault on the ball makes with the races in one revolution of the shaft. This peak will also be non-synchronous and is called a bearing tone. The fault on the ball or roller also travels in and out of the load zone, however it travels at the cage rate, not the shaft rate. Therefore, the sideband spacing around the bearing tone will be equal to the cage rate, which is usually in the neighborhood of 0.3x the shaft rate.

Conclusion

The concept of linear and non-linear behavior gives us another way to think about a vibration spectrum and how its appearance relates to machine faults. Healthy machines should respond more linearly than machines with faults, which is to say, as machines develop faults they will likely respond less linearly. As they become less linear we begin to see more and larger harmonics and or sidebands in our spectra.

Because we may not know all of the details about the design of a machine or how its spectra will appear when it is healthy, it is still best to trend information over time. This is to say, look for more and larger harmonics and new peaks that were not there before as an indication that the health of the machine is deteriorating.

About the author:
In eleven years at DLI Engineering, Alan Friedman has worked in software development, expert system development, data analysis, training, and installation of predictive maintenance programs. He is a graduate of Tufts University with a B.S. in mechanical engineering.
***Special thanks** to Glenn White!*